Functions and Graphs

Harder Polar and parametric graphs

Some graphs are difficult to draw. You may use some software to help you.

e.g. Winplot (http://math.exeter.edu/rparris/) , GRAPES (http://www.criced.tsukuba.ac.jp/grapes/)

1. Sketch :

(a) $\rho^2 = a^2 \cos 2\theta$ **(b)** $\rho = a(1 - \cos\theta)$ (Cardoid)

(c) $\rho = a\theta$ (Spiral of Archimedes) **(d)** $\rho = e^{a\theta}$ (Logarithmic spiral)

(e) $\rho = a \sin 3\theta$ (Rose of 3 leaves) **(f)** $\rho = a \sin 2\theta$ (Rose of 2 leaves)

(g) $\rho = a \sin\frac{\theta}{2}$ **(h)** $\rho = b - a\cos\theta \quad (b < a)$ (Limacon)

(i) $\rho^2\theta = a^2$ (Lituus) **(j)** $\rho\theta = a$ (Hyperbolic spiral)

(k) $\rho = \sec\theta \pm a$ (Concoid of Nicomedes)

2. Sketch the curve $\rho = 2a(1 + \cos\theta)$. Find the polar coordinates of the points in which the curve meets the line $2\rho\cos\theta + a = 0$.

3. Plot the graphs of

(a) $x = 2r\cos\theta + r\cos 2\theta$, $y = 2r\sin\theta - r\sin 2\theta$ (Hypo-cycloid of 3 cusps)

(b) $x = \frac{3at}{1+t^3}$, $y = \frac{3at^2}{1+t^3}$ (Folium of Descartes)

4. By using the given parametrization, plot the graphs of the followings :

(a) (Cissoid of Diocles) $y^2(2a - x) = x^3$ $(y = tx)$

(b) (Hypocycloid of 4 cusps) $x^{2/3} + y^{2/3} = a^{2/3}$ $(x = a\sin^3\theta)$

5. Plot the following graphs :

(a) $\rho = \cos 7\theta + 3$ **(b)** $\rho = \cos\frac{9}{10}\theta$ **(c)** $\rho = \cos\frac{3}{10}\theta$

(d) $\rho = \cos\frac{9}{4}\theta \quad -\frac{1}{3}$ **(e)** $\rho = a(\cos\theta + 1)$ **(f)** $\rho = a\left(\cos\frac{7\theta}{2} + 1\right)$

(g) $\rho = a\left(\cos\frac{7\theta}{2}\right)$ **(h)** $\rho = a\left(\cos 2\theta + \frac{1}{3}\right)$ **(i)** $\rho = a\left(\cos\frac{7\theta}{2} + 4\right)$